

Fig. 4. Bias dependence of the  $C_{be}$ ,  $\tau$ , and  $\beta_0$  insignificances using  $S$ -parameters only ( $F_1-F_4$ ) and using the  $S$ -parameters and the current gain term  $B$  ( $F_1-F_5$ ).

## V. CONCLUSIONS

Several techniques for numerical parameter extraction have been evaluated. The significance of small-signal element values extracted by numerical fitting to measured  $S$ -parameters was considerably better than fitting to  $Z$ -parameters and was further improved by adding a function that emphasizes the most unreliable elements. The chosen  $B$ -function does not need to be the best choice, but it also significantly supports the physical relevance of these elements, thereby improving the usefulness of the extracted model for process monitoring.

## REFERENCES

- [1] D. R. Pehlke and D. Pavlidis, "Evaluation of the factors determining HBT high-frequency performance by direct analysis of  $S$ -parameter data," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 2367–2373, Dec. 1992.
- [2] S. J. Spiegel, D. Ritter, R. A. Hamm, A. Feygenson, and P. R. Smith, "Extraction of the InP/GaInAs heterojunction bipolar transistor small-signal equivalent circuit," *IEEE Trans. Electron Devices*, vol. 42, pp. 1059–1064, June 1995.
- [3] J. M. M. Rios, L. M. Lunardi, C. S., and Y. Miyamoto, "A self-consistent method for complete small-signal parameter extraction of InP-based heterojunction bipolar transistors (HBT's)," *IEEE Trans. Microwave Theory Tech.*, vol. 45, pp. 39–44, Jan. 1997.

- [4] S. Tiwari, *Compound Semiconductor Device Physics*. New York: Academic, 1992.
- [5] S. A. Maas and D. Tait, "Parameter-extraction method for heterojunction bipolar transistors," *IEEE Microwave Guided Wave Lett.*, vol. 2, pp. 502–504, Dec. 1992.
- [6] I. Schnyder, M. Rohner, D. Huber, C. Bergamaschi, and H. Jäckel, "A laterally etched collector InP/InGaAs(P) DHBT process for high speed power applications," presented at the Int. Conf. Indium Phosphide and Related Mater., 2000.

## Comments on Representation of Surface Leaky Waves on Uniplanar Transmission Lines

Jan Macháč and Ján Zehentner

**Abstract**—This paper compares partial waves approximating a surface leaky wave on a uniplanar transmission line with substrate surface waves supported by its substrate. It is shown that their field distributions and the propagation constants differ. These differences increase with rising leakage constant, and with the growing distance of the corresponding pole of the Green function from the real axis on the spectral variable complex plane. The reported findings are demonstrated on the slotline.

**Index Terms**—Leaky waves, printed-circuit lines, slotlines.

## I. INTRODUCTION

Leaky waves considerably deteriorate the behavior of planar microwave and millimeter-wave circuits and transmission lines due to increased losses, occurrence of crosstalk between neighboring parts of the circuit, and pulse distortion. For these reasons, leaky waves have been intensively studied in recent years [1]–[4]. There are two kinds of leaky waves supported by open planar transmission lines. Surface leaky waves take power away into the dielectric substrate, while space leaky waves radiate into space and may also leak power into the substrate. In this paper, we discuss only surface leaky waves.

There is a general understanding that partly or entirely open planar transmission lines can suffer from loss of transmitted power leaking into the surface leaky waves, which, far away from the line axis, pass on the TM or TE waves supported by the dielectric substrate [1], [2], [5]. A leaky wave can be interpreted as a superposition of two or more nonuniform partial waves propagating at some angle to the line axis, plus the remaining field bound to the line [2], [5], [6]. The propagation constants of these partial waves are assumed to be equal to the propagation constant of the substrate surface wave  $k_s$  [5], which is real for a lossless line. Assuming sufficiently weak leakage, when the imaginary part of a leaky-wave propagation constant is negligible in comparison with phase constant  $\beta$ , the angle under which the partial wave propagates is [2], [3]

$$\Theta_\beta = \arccos \frac{\beta}{k_s}. \quad (1)$$

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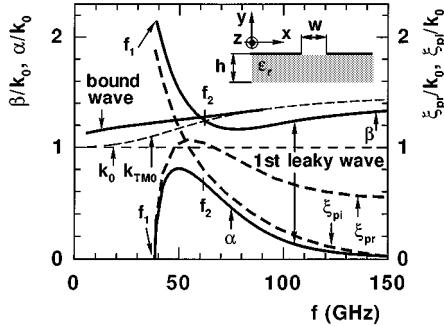


Fig. 1. Dispersion characteristics and the Green function pole position of the lossless slotline when  $w = h = 1$  mm and  $\epsilon_r = 2.25$ .

A comparison of the partial waves approximating the leaky wave with the substrate surface waves made in this paper reveals that these waves generally differ from each other, both in the propagation constant and field distribution. The stronger the leakage is, the higher this difference. The presented conclusions, which are not in conformity with the concept of power leakage into substrate surface waves [1], [2], [5], follow from an investigation of leaky-wave propagation on the slotline.

## II. LEAKY-WAVE REPRESENTATION

First, let us recall and then extend surface leaky-wave representation on an open planar transmission line, e.g., on the slotline. The slotline, the cross section of which is shown in the inset of Fig. 1, has been investigated by the spectral-domain method. The normalized phase  $\beta/k_0$  and leaky  $\alpha/k_0$  constants of the bound and first leaky waves are shown in Fig. 1, together with the position of the normalized pole of the Green function  $\xi_p/k_0 = \xi_{pr}/k_0 + j \xi_{pi}/k_0$  [6]. The  $TM_0$  surface-wave propagation constant is  $k_{TM_0}$ , and  $k_0$  denotes the free-space wavenumber.

According to [2], [5], and [6], the two nonuniform partial waves given by the  $y$ -oriented  $E$ -field components  $E_{yR}$  and  $E_{yL}$  on the substrate surface where  $y = 0$  represent the field of the first leaky wave far away from the slot. Accordingly,

$$E_{yR}(x, z) = j \text{Res}(-\xi_p, 0) e^{-j\xi_p x} e^{-j\gamma z} \quad (2)$$

propagates obliquely to the right-hand side from the slot, and

$$E_{yL}(x, z) = -j \text{Res}(\xi_p, 0) e^{j\xi_p x} e^{-j\gamma z} \quad (3)$$

propagates at the same angle to the left-hand side from the slot.  $\text{Res}(\pm\xi_p, y)$  is the residuum of the Fourier transform of field  $E_y$  [6, eq. (4)] at the pole  $\pm\xi_p$  and at position  $y$ . The propagation constant along the line  $\gamma = \beta - j\alpha$ , i.e., in the  $z$ -direction, is complex.

The partial-wave field distribution, together with the corresponding wave vectors, is sketched in Fig. 2. At the beginning, a lossy substrate is assumed. The partial-wave propagation constant in the  $x$ -direction is determined by the complex pole  $\xi_p$  so that  $k_x = \xi_p$ . Both constants  $\xi_p$  and  $\gamma$  are coupled with the propagation constant of the corresponding surface wave  $k_s = k_{sr} + j k_{si}$ , which is either  $k_{TM}$  or  $k_{TE}$ , [7], [5], [6] and, in this manner,

$$k_s^2 = \gamma^2 + \xi_p^2. \quad (4)$$

Inserting complex values of  $\xi_p = \xi_{pr} + j \xi_{pi}$  and  $\gamma = \beta - j\alpha$  into (2), we get

$$E_{yR}(x, z) = j \text{Res}(-\xi_p, 0) e^{-(\alpha z - \xi_{pi} x)} e^{-j(\beta z + \xi_{pr} x)} \\ = j \text{Res}(-\xi_p, 0) e^{\alpha \mathbf{p} \cdot \mathbf{r}} e^{-j\beta \mathbf{p} \cdot \mathbf{r}} \quad (5)$$

where  $\alpha \mathbf{p} = x_0 \xi_{pi} - z_0 \alpha$  and  $\beta \mathbf{p} = x_0 \xi_{pr} + z_0 \beta$  are the attenuation and phase vectors of the partial wave, respectively, shown in Fig. 2, and

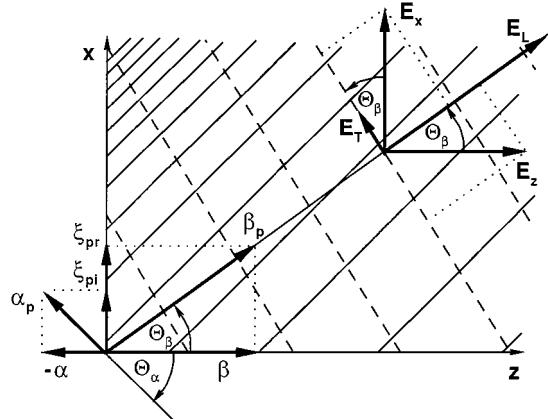


Fig. 2. Sketch of the leaky-wave field distribution approximated by the nonuniform wave (2) represented by solid lines of constant amplitude and by dashed lines of constant phase together with the decomposition of the wave and the field vectors. The angle  $\Theta_\beta$  represents  $\Theta_\beta^{(7)}$  defined in (7).

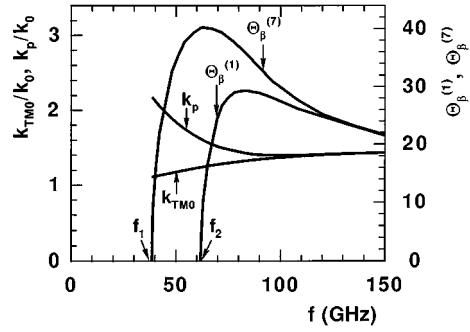


Fig. 3. Phase constant and angle of propagation of the partial waves approximating the first leaky wave on the slotline from Fig. 1.

$\mathbf{r} = x_0 \mathbf{x} + z_0 \mathbf{z}$ . The modulus of  $\beta_p$  determines the partial-wave phase constant  $k_p$ . Using (4), we get

$$k_p = |\beta_p| = \sqrt{\beta^2 + \xi_{pr}^2} = \sqrt{k_{sr}^2 - k_{si}^2 + \alpha^2 + \xi_{pi}^2} \quad (6)$$

which corresponds to [7, eq. (16-8c)]. Now the direction of the phase vector  $\beta_p$  in Fig. 2 determining the propagation direction of the partial wave is, in accordance with [5], defined by the angle

$$\Theta_\beta = \arctan \frac{\xi_{pr}}{\beta} = \arccos \frac{\beta}{k_p} = \arcsin \frac{\xi_{pr}}{k_p}. \quad (7)$$

Similarly, the direction of  $\alpha_p$ , in which the amplitude of the partial wave grows, determines angle  $\Theta_\alpha$

$$\Theta_\alpha = \arctan \left( -\frac{\xi_{pi}}{\alpha} \right). \quad (8)$$

The values of angles  $\Theta_\beta^{(1)}$ , defined by (1), and  $\Theta_\beta^{(7)}$ , defined by (7), are plotted in Fig. 3 for the lossless line from Fig. 1. These angles are generally different. At frequency  $f_1$ , where the solution of the dispersion equation starts to be complex (Fig. 1), the Green function pole lies on the imaginary axis of the spectral variable complex plane and  $\alpha = 0$ . Equation (6) now provides  $k_p = \beta$  and from (7)  $\Theta_\beta^{(7)} = 0$ . On the other hand, from (1), we get  $\Theta_\beta^{(1)} = 0$  at frequency  $f_2$ , where the leaky wave starts to be physical. Between frequencies  $f_1$  and  $f_2$ , the first leaky wave is slow with respect to the  $TM_0$  surface wave, as  $\beta \geq k_{TM_0}$ . Such a wave cannot radiate into the substrate and is nonphysical. The extent to which this wave is physically meaningful must be determined by examining the total field produced by an actual source [4]. The functions describing the field distribution, however, also exist in the frequency range from  $f_1$  to  $f_2$ , and the nonphysical angle of

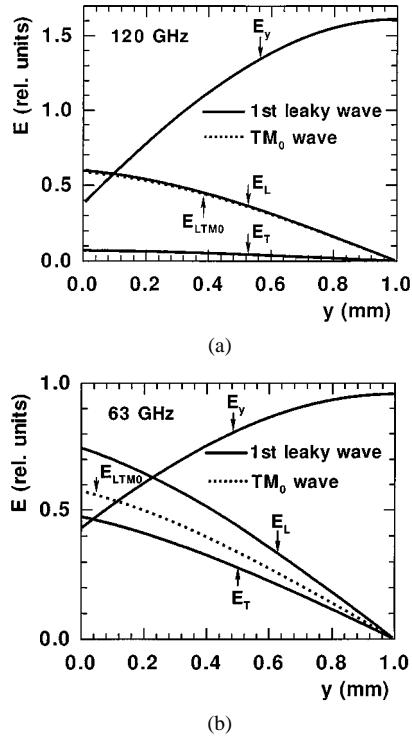


Fig. 4. Field distribution of the first leaky wave and the  $\text{TM}_0$  surface wave across the substrate at the point  $x = 7.5$  mm,  $z = 0$  mm and at a frequency of: (a) 120 GHz and (b) 63 GHz.  $E_{LTMO}$  denotes the  $E_L$ -field component of the  $\text{TM}_0$  surface wave.

propagation of the wave  $\Theta_\beta^{(7)} \neq 0$ . The exact calculation of the field distribution confirms that lines of constant phase far from the slot axis are really determined by angle  $\Theta_\beta^{(7)}$ . At sufficiently higher frequencies than  $f_2$ , the two values  $\Theta_\beta^{(1)}$  and  $\Theta_\beta^{(7)}$  are nearly identical (see Fig. 3).

### III. COMPARISON OF LEAKY AND SURFACE WAVES

Fig. 3 shows that the partial-wave phase constant  $k_p$  (6) generally differs from the propagation constant of the corresponding surface wave  $k_{sr}$  represented here by  $k_{\text{TM}_0}$ . It is evident that, the higher the magnitudes of  $\xi_{pi}$  and  $\alpha$  are in comparison with  $\beta$ , the greater the difference between  $k_p$  and  $k_{sr}$ . This means that the difference is more significant in the case of more intensive leakage.

Let us now compare field distributions of the first leaky wave approximated by the partial wave and the substrate surface wave propagating in the same direction. A grounded substrate supports a TM surface wave with an  $E_y$ -field component perpendicular to the substrate, and a transversal component  $H_T$  and a longitudinal component  $E_L$  both parallel to the substrate surface [8]. Conversely, a TE wave has nonzero  $H_y$ ,  $H_L$ , and  $E_T$ -field components. According to Fig. 2, the longitudinal and transversal components of the partial wave can be written by means of  $E_x$ - and  $E_z$ -field components

$$E_L = E_z \cos \Theta_\beta + E_x \sin \Theta_\beta \quad (9)$$

$$E_T = E_x \cos \Theta_\beta - E_z \sin \Theta_\beta. \quad (10)$$

$H_T$ - and  $H_L$ -field components can be expressed similarly. Therefore, the partial wave has, with respect to its direction of propagation, the field components  $E_y$ ,  $E_L$ ,  $H_T$ ,  $E_T$ ,  $H_L$ . It differs from the  $\text{TM}_0$  surface wave due to the presence of  $E_T$  and  $H_L$ . The higher  $\xi_{pi}$  and  $\alpha$  are, the more remarkable the difference of these two wave fields. This is illustrated in Fig. 4, where the first leaky-wave field computed exactly by the backward Fourier transformation [6] at the point  $x = 7.5$

mm and  $z = 0$  mm is compared with the  $\text{TM}_0$  surface-wave field [8]. The fields are recalculated in such a manner that  $E_y$  has the same magnitude for both waves. The lossless slotline from Fig. 1 serves for demonstration. The field in Fig. 4(a) is plotted for  $f = 120$  GHz, where  $\gamma = 3215.77 - j 182.44 \text{ m}^{-1}$ ,  $\xi_p = 1507.63 + j 389.12 \text{ m}^{-1}$  and  $k_{\text{TM}_0} = k_{sr} = 3525.54 \text{ m}^{-1}$ . Accordingly,  $\alpha \ll \beta$ ,  $\xi_{pi} < \beta$ , and the ratio  $|E_L|/|E_T| = 8.23$ . Thus,  $E_T$  is small in comparison to  $E_L$ . The distributions of the  $E_L$  fields of the first leaky wave and the  $\text{TM}_0$  surface wave are almost equal. Therefore, the partial wave at 120 GHz corresponds to the  $\text{TM}_0$  surface wave. A different situation is at 63 GHz, just above frequency  $f_2$  (Fig. 1). The  $E_L$ -field components of both the partial and surface waves considerably differ and the  $E_T$ -field component of the partial wave cannot be neglected at this frequency [see Fig. 4(b)].

Conclusions derived here for the first leaky wave are also valid for the second and higher leaky waves. In such cases, the leaky wave should be decomposed into a corresponding number of partial waves [6], which ought to be compared separately with either TM or TE surface waves.

### IV. CONCLUSIONS

Partial waves representing the surface leaky wave far from the line axis have been compared with the substrate surface wave propagating in the substrate in the same direction as the partial wave.

Until now, the literature has commonly stated that the leakage of power from open uniplanar transmission lines goes into  $\text{TM}_0$ ,  $\text{TE}_1$  and contingently into the higher substrate surface waves. Our analysis concludes that this is the case for weak leakage only, when the leakage constant and the Green function pole distance from the real axis of the spectral variable complex plane are small with respect to the leaky-wave phase constant. Concerning the first leaky wave on the slotline, this takes place at high frequencies only. At frequencies just above the leakage cutoff, the partial waves approximating the leaky wave differ remarkably from the corresponding substrate surface wave, both in the propagation constant and field distribution. Consequently, it is not possible to generalize that power leaks into substrate surface waves. Measurement of the angle of propagation of a partial wave above, but close to frequency  $f_2$ , when  $\Theta_\beta^{(1)}$  and  $\Theta_\beta^{(7)}$  are sufficiently different, could provide confirmation of these findings.

The conclusions given above result from slotline analysis, and are valid for both lossless and lossy lines. They are also applicable to the surface leaky waves on other uniplanar transmission lines.

### REFERENCES

- [1] A. A. Oliner, "Leakage from various waveguides in millimeter wave circuits," *Radio Sci.*, vol. 22, pp. 866–872, Nov. 1987.
- [2] N. K. Das and D. M. Pozar, "Full-wave spectral-domain computation of material, radiation, and guided wave losses in infinite multilayered printed transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 54–63, Jan. 1991.
- [3] H. Shigesawa, M. Tsuji, and A. A. Oliner, "Dominant mode power leakage from printed-circuit waveguides," *Radio Sci.*, vol. 26, pp. 559–564, Mar./Apr. 1991.
- [4] C. Di Nallo, F. Mesa, and D. R. Jackson, "Excitation of leaky modes on multilayer stripline structures," *IEEE Trans. Microwave Theory Tech.*, vol. 46, pp. 1062–1071, Aug. 1998.
- [5] F. Messa and R. Marqués, "Power based considerations in the spectral domain analysis of leaky waves in covered strip-like transmission lines," *Proc. Inst. Elect. Eng.*, vol. 143, no. 1, pp. 25–30, Feb. 1996.
- [6] J. Zehntner, J. Macháč, and M. Migliozzi, "Upper cutoff frequency of the bound wave and new leaky wave on the slotline," *IEEE Trans. Microwave Theory Tech.*, vol. 46, pp. 378–386, Apr. 1998.
- [7] F. J. Zucker, "Surface- and leaky-wave antennas," in *Antenna Engineering Handbook*, H. Jasik, Ed. New York: McGraw-Hill, 1961.
- [8] R. E. Collin, *Field Theory of Guided Waves*, 2nd ed. New York: IEEE Press, 1991.